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Prolongations of tensor fields and connections to tangent bundles II

—Infinitesimal automorphisms—

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1. Introduction.

In our previous paper [3] we defined the notion of complete lift. It is a natural way to prolong tensor fields and connections of a manifold M to the tangent bundle $T(M)$. Referring the reader to our previous paper [3] for necessary notations and terminologies, we state our main result of the present paper.

THEOREM 1. *Let* ∇ *be a torsionfree affine connection on a manifold* M *and* ∇^c *its complete life to* $T(M)$. *Let* X *and* Y *be infinitesimal affine transformations of* M *and* U *a parallel tensor field of type* (1, 1) *on* M *such that*

$$U \circ R(Z, W) = R(UZ, W) = R(Z, UW) = R(Z, W) \circ U \tag{1}$$

for all vector fields Z, W *of* M,

where R *denotes the curvature tensor field of* ∇. *Then* $X^c + Y^v + \iota U$ *is an infinitesimal affine transformation of* $T(M)$.

Conversely, every infinitesimal affine transformation of $T(M)$ *may be uniquely written as* $X^c + Y^v + \iota U$, *where* X, Y *and* U *are as above, if* M *does not admit a nonzero parallel tensor field* A *of type* (1, 1) *such that*

$$A \circ R(Z, W) = R(AZ, W) = R(Z, AW) = R(Z, W) \circ A = 0 \tag{2}$$

for all vector fields Z, W *of* M.

The following facts will be also shown.

REMARK 1. In any of the following cases M does not admit a nonzero parallel tensor field A of type (1, 1) satisfying (2):

(a) M is non-flat and the linear holonomy group of M is irreducible;

(b) M is Riemannian and has no Euclidean factor in its de Rham decomposition,

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