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On discrete subgroups of the two by two projective linear group over $\mathfrak{p}$-adic fields

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Introduction. In the present paper we shall prove some properties of torsion-free discrete subgroups Γ of the title which were announced in our previous note [2], and then we shall show a method for the construction of all such Γ. Those properties are possessed by subgroups Γ of more general abstract groups G (defined in § 1), e. g. free product of two groups with some amalgamated subgroups, and so we shall treat them together in an abstract manner. In § 2, we shall show that Γ is isomorphic to a *free group* with some explicitly given set of generators. In § 3, we shall compute the number of primitive conjugacy classes of Γ with given "degree" or, what is the same, evaluate certain "ζ function" attached to $\Gamma \subset G$. This is based on the results of § 2. § 4 is for the *construction of all* Γ. The problem is nothing but a purely combinatorial one. There are many Γ and they have many non-trivial deformations. In the case where $G = PL(2)$ over $\mathfrak{p}$-adic fields, these, together with the remarks on spectral decompositions of $L^2(G/\Gamma)$ given at the end of § 3, show that although some Γ (with G/Γ compact) are arithmetically defined, their arithmetical properties are not preserved by taking subgroups with finite indices (cf. also [2] § 4); here everything is algebraic and, in general, not arithmetic. For example, Ramanujan's conjecture for some type of modular cusp forms is equivalent with some conjecture for arithmetically defined Γ, but the latter fails to be true if we take some subgroups of Γ with finite indices instead of Γ. Finally in § 5, a remark on the structure of "$\mathfrak{p}$-unit groups" of totally definite quaternion algebras, which is a direct application of Theorem 1 (§2), is given.

Throughout the followings, for any set S, $|S|$ will denote its cardinal number; and the summation symbol Σ over some subsets of a set implies disjoint union. For any ring A and positive integer n, $M(n, A)$ will denote the ring of all n by n matrices whose entries are elements of A.

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