

Prolongations of tensor fields and connections to tangent bundles I — General theory —

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1. Introduction and notations.

The purpose of this paper is to define a natural derivational mapping (called the complete lift¹⁾) of the algebra $\mathcal{T}(M)$ of tensor fields of a manifold M into the algebra $\mathcal{T}(T(M))$ of tensor fields of the tangent bundle $T(M)$ of M , to associate with each affine connection ∇ of M an affine connection ∇^c (called the complete lift of ∇) of $T(M)$ in a natural way and to derive basic formulas and properties of the complete lift. To define the notion of complete lift, we introduce also that of vertical lift and transvection as well as a more familiar notion of Lie derivation. The notions of complete lift and vertical lift have been already defined for tensor fields of special kinds by several authors, [5], [7], [8], [9], [14], [15].

Using the notion of complete lift we shall show that such familiar G -structures as a pseudo-Riemannian structure, an almost complex structure and a symplectic structure on M induce similar structures on the tangent bundle $T(M)$. An unexpected but perhaps more interesting result is that each pseudo-Riemannian (resp. affine) symmetric space structure on M induces a pseudo-Riemannian (resp. affine) symmetric space structure on $T(M)$. This suggests us a method of producing a large class of affine symmetric spaces. Let A be a local algebra of the form $A=R+I$ where R is the field of real numbers and I is the maximal ideal of A such that $\dim I < \infty$ and $I^k=0$ for some k . Weil has shown [10] that such an algebra A defines a fibre bundle $A(M)$ over M , generalizing the construction of the tangent bundle $T(M)$. (If $\dim I=1$ and $I^2=0$, then $A(M)$ is nothing but the tangent bundle $T(M)$ of M .) A successful generalization of our theory to $A(M)$ would furnish a useful tool for the differential geometry of higher order contact and yield a large number of affine symmetric spaces.

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1) Perhaps, "natural lift" is more appropriate. But in conformity with other authors, we use the term "complete lift".