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On the automorphism group of a G-structure

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§ 0. Introduction.

A linear Lie group is called *elliptic* if its Lie algebra contains no matrix of rank one. A G-structure is called *elliptic* if G is *elliptic*. (N. B. G is a linear subgroup of $GL(n, \boldsymbol{R})$.) The purpose of this paper is to prove that the globally defined infinitesimal automorphisms of a G-structure (called G-vector field) are given by a system of linear elliptic differential equations if and only if this G-structure is elliptic. (See Lemma for a precise statement.) It follows easily

THEOREM A. *The group of diffeomorphisms of M which leave a given elliptic G-structure invariant is a finite dimensional Lie group, provided M is compact.*

Theorem A is a generalization of the results of Boothby-Kobayashi-Wang [1] and Ruh [8]. (In fact, Ruh's sufficient condition clearly implies that the G-structure in question is elliptic.) Both Lemma and Theorem A are contained implicitly in Guillemin-Sternberg [3]. Still we feel their explicit statements with proofs would be worth publishing because of their importance. Also we shall provide two examples to show that Theorem A is best possible in a sense, following suggestions of Professor S. Kobayashi and Professor S. Sternberg. Also the author wishes to express his thanks to Professor T. Nagano and Professor M. Kuranishi.

§ 1. Let $P(M, \pi, G)$ be any G-structure on M, and $\mathfrak{g}$ be the Lie algebra of G. That is, P is a subbundle with structure group G of the frame bundle of M. A (local) diffeomorphism of M is a *(local) G-automorphism* if and only if it leaves the G-structure $P(M, \pi, G)$ invariant.

Let $\{x^1, \cdots, x^n\}$ be a local coordinate system around $z \in M$, defined on an open neighbourhood U of M. Furthermore, we assume that the neighbourhood U is so small that it admits a local cross-section ϕ from U into P. Let V be an open set of U. A local diffeomorphism f from V into U is a local G-automorphism if and only if there exists a mapping g from V into G such that

(1) $$(df)(\phi(x)) = \phi(f(x)) \cdot g(x)$$