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On Mordell's conjecture for algebraic curves over function fields

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Introduction.

In this paper, we are concerned with Mordell's conjecture on the set of rational points on algebraic curves in "relative case" (cf. [2] p. 139).

Let k be any field and K be a function field with k as constant field, i.e. a regular extension of finite type of k. Let C be a complete non-singular curve defined over K. We say that C is *trivially defined*, if there is a curve C_0 defined over k which is birationally equivalent to C over K. Then our main Theorem reads:

*If the genus g of C is $\geqq 2$, then the set of all rational points of C over K is a finite set or C is trivially defined**).

This was proved by Grauert [3] in the case where the characteristic of k is 0 and k is algebraically closed. Manin [4] obtained the same result with a transcendental method. We shall prove the above Theorem for the field k of any characteristic p (which may be $=0$ or $\neq 0$), without supposing k to be algebraically closed.

The proof is given in two cases (1) $p=0$ (§ 1), (2) $p \neq 0$ (§ 2)[1]. We shall use the results of [3] as formulated at the beginnings of § 1 and § 2, and the theory of abelian varieties (cf. [1], [6]). As to the terminology we follow generally the usage in [1].

More specifically, the method we shall use is that of descent. To explain

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*) For the case $p=0$, we shall prove another related proposition concerning the curve of genus 1. (See Proposition 1 below.)

1) To avoid misunderstanding we add here the following remark. Grauert [3] introduced the notion of "quasi-trivially defined curve" which implies that of "trivially defined curve" when $p=0$. He considered also a certain fibre variety X with C as fibre, such that when X becomes trivial (i.e. isomorphic to the direct product of fibre and base space), then C is trivially defined in our sense. He proved that in case $p=0$, X becomes trivial when C is quasi-trivial and used this to obtain his main Theorem. For the case $p \neq 0$, he constructed an example showing that X need not become trivial even if C is quasi-trivial. But this is of course in no contradiction with the validity of our Theorem for $p=0$.