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On the continuation of analytic sets

By Hirotaka FUJIMOTO

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§ 1. Introduction.

1. It is well known as Hartogs-Osgood's theorem that for a relatively compact domain D in C^n $(n \geqq 2)$ with the connected boundary ∂D every holomorphic function in a connected neighborhood of ∂D is continuable to D. In [**21**], Rothstein gave an analogous continuation theorem of analytic sets in domains in C^n with suitable convexity conditions. In this paper, we attempt to generalize his results to the case of analytic sets in complex spaces[1].

As in the proof of Hartogs-Osgood's theorem [**7**], we consider a real-valued function v such that for any p an analytic set M in $\{v > v(p)\}$ is continuable to a neighborhood of p (local continuability) and assert $\inf \Lambda = -\infty$ for the set Λ of all λ satisfying that M is continuable to $\{v > \lambda\}$ (global continuability). For the study of local continuability, Rothstein restricted himself to the case of grad $v \neq 0$. His results are insufficient for the study of the global continuability of analytic sets in complex spaces. With some improvements in his arguments, we shall first prove the following local continuation theorem.

If an open set D in a complex space X is $*$-strongly s-concave at p in X (see Definition 2.8), every purely $(s+1)$-dimensional analytic set in D is continuable to a neighborhood of p.

The first four sections are devoted to the proof of this theorem. In § 2, we define several kinds of convex functions and convex open sets in a complex space and give some elementary properties and the relations of these convexities. In § 3, we state the definition of the continuation of analytic sets in order to avoid misuse and ambiguity of terminology. As in case of holomorphic functions, we need the theorem of identity for irreducible analytic sets. Using this, we give some general properties on the continuation of analytic sets (§ 3, 2° and 3°). The proof of the above local continuation theorem can be reduced to the study of a special complex space X which is

1) In this paper, a complex space means "β-Raum" in the sense of Grauert-Remmert [**10**] and is always assumed to be σ-compact.