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On compact complex analytic manifolds of complex dimension 3.

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The purpose of this paper is to prove some analogous propositions to the results of Kodaira [8] in three dimensional case. Terminologies and notations are the same as those in Kodaira [8]. We shall use the fundamental results of Hironaka [5].

Let M^n be a compact complex analytic manifold of complex dimension n. Let $\mathfrak{F}(M^n)$ be the field of all meromorphic functions on M^n. Then by a theorem of Chow-Remmert [**9**] $\mathfrak{F}(M^n)$ is an algebraic function field of complex dimension not greater than n. Hence there is a non-singular projective model V of $\mathfrak{F}(M^n)$. We identify $\mathfrak{F}(M^n)$ and the function field of V. Let $(1, x^1, \cdots, x^\nu)$ be a generic point of V. Then $x^i \in \mathfrak{F}(M^n)$. Hence we obtain a mapping

$$\Phi: \quad M \ni z \rightarrow (1, x^1(z), \cdots, x^\nu(z)) \in V.$$

PROPOSITION. *Φ is a meromorphic mapping. That is, there exists an irreducible and locally irreducible complex subspace X of $M^n \times V$ which is the closure of the graph of Φ and the natural projection p of X to M^n is a proper modification.*

$$\begin{array}{ll} \varphi: & X \xrightarrow{\ \iota\ } M^n \times V \longrightarrow V \\ & \quad \searrow_{p} \quad \downarrow \\ & \qquad\quad M^n \end{array}$$

Proof is parallel to Remmert [**10**] and we do not reproduce it here.

Let φ be the natural projection from X to the second component V.

Clearly the underlying continuous map of φ is surjective and φ induces an isomorphism of $\mathfrak{F}(X)$ and $\mathfrak{F}(V)$, where $\mathfrak{F}(X)$ and $\mathfrak{F}(V)$ are the function fields of X and V, respectively.

THEOREM 1. *Every fibre of φ is connected. Consequently, if* $\dim \mathfrak{F}(M^n)=n$, *then M^n is bimeromorphically equivalent to a non-singular projective variety.*

COROLLARY. *If* $\dim \mathfrak{F}(M^n)=n=3$, *then the first Betti number of M^3 is even.*

Let n be equal to 3 and $\rho: M' \rightarrow X$ be the resolution of singularities.