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On the variety of orbits with respect to an algebraic group of birational transformations

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For an algebraic variety V and an algebraic group G operating on V, we can construct the variety V_G of G-orbits on V and the natural rational mapping f of V to V_G (cf. [8]). The variety V_G is obtained as a model of the subfield of all the G-invariant elements in the field of rational functions on V.

The purpose of this paper is to prove several results concerning on the relations between the Albanese varieties (and the spaces of linear differential forms of the first kind) of V and of V_G. Denoting by G_0 the connected component of G containing the identity element, we see that the finite group G/G_0 operates on the variety V_{G_0} of G_0-orbits on V and V_G is naturally birationally equivalent to the variety $(V_{G_0})_{G/G_0}$ of (G/G_0)-orbits on V_{G_0}. Hence we may restrict ourselves to the two cases: (i) G is connected and (ii) G is a finite group; and the second case (ii) has already been treated in our previous paper [3].

In § 1, we shall give the definition of the variety V_G and prove several preliminary results.

In § 2, we shall first construct the Albanese variety $\mathrm{Alb}(V_G)$[1] of V_G as a quotient abelian variety of the Albanese variety $A=\mathrm{Alb}(V)$ of V (Theorem 1). In particular, for the connected algebraic group G_0, we define a rational homomorphism φ of G_0 into A and it will be proved that $A_1=A/\varphi(G_0)$ is the Albanese variety of V_{G_0} (Theorem 2). Then we shall also prove that $\mathrm{Alb}(V)$ is isogenous to the direct product of $\mathrm{Alb}(V_{G_0})$ and the Albanese variety of the generic G_0-orbit $\overline{G_0P}$[2] on V (Theorem 3) and we have the inequality $0\leqq \dim \mathrm{Alb}(V)-\dim \mathrm{Alb}(V_{G_0})\leqq \dim V-\dim V_{G_0}$. Moreover, by means of the l-adic representations $M_l^{(A)}$ and $M_l^{(A^*)}$ of the rings of endomorphisms of A and $A^*=\mathrm{Alb}(G_0)$, we define the two matrix representations of the finite group G/G_0. Then, if G operates regularly and effectively on V, we shall show that the dimension of $\mathrm{Alb}(V_G)$ is equal to the half of the difference of the multi-

1) For a variety W, $\mathrm{Alb}(W)$ denotes an Albanese variety of W.
2) Cf. § 1.