

On the ζ -functions of a total matrix algebra over the field of rational numbers

By Moto-o KINOSHITA

(Received Jan. 28, 1965)

Introduction

Iwasawa and Tate [8, 15] reconstructed the theory of Hecke's L -function as a theory of the ζ -function, attached to a number field k , with a character of the idele class group of k . Since then, it has been expected to generalize this theory to the case of the ζ -function of a simple algebra over the rational number field \mathbf{Q} . Let A and G be the adèle ring and the idele group of a simple algebra over \mathbf{Q} respectively. Fujisaki [2, 3] solved the problem for the ζ -function with an abelian character of G . The theory of Fujisaki includes the results of Hey and Eichler [7, 1]. Godement [4] showed the possibility of applying the Iwasawa-Tate method to the ζ -function, attached to a division algebra, with a "non-abelian character" of G . Tamagawa [14] developed the theory of Euler product. He determined an explicit form of the local ζ -function, attached to a simple algebra, with a zonal spherical function. And he proved that the ζ -function of a division algebra, defined as an infinite product of local ζ -functions, satisfies a functional equation. From the theory of Maass [9] on the Dirichlet series corresponding to a non-holomorphic automorphic function on upper half-plane, we can extract a theory of the ζ -function, attached to the total matrix algebra of degree 2 over \mathbf{Q} , with a zonal spherical function.

On the other hand, Hecke [6] gave the theory of constructing Dirichlet series with Euler product and functional equation out of a modular form. Shimura [12] generalized this theory to the case of the automorphic form of Hilbert type by means of the Iwasawa-Tate method. In other words, the ζ -function of a quaternion algebra, with a spherical function, not necessarily of class 1, was treated.

The purpose of the present paper is to prove that the ζ -function of a total matrix algebra over \mathbf{Q} is defined as an infinite product of local ζ -functions, is meromorphic on the whole z -plane and satisfies a functional equation, if the "character" is a zonal spherical function determined by a certain automorphic function on G (cf. § 6, Theorem).