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Closed hypersurfaces with constant mean curvature in a Riemannian manifold

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It has been proved by H. Liebmann [3] and W. Süss [4] that the only convex closed hypersurface with constant mean curvature is a sphere. To prove this theorem we need integral formulas of Minkowski.

Prof. Y. Katsurada [1], [2] derived integral formulas of Minkowski type which are valid in an Einstein space and proved the following generalisation of the theorem of Liebmann-Süss.

THEOREM. *Let M be an $(m+1)$-dimensional orientable Einstein space and S a closed orientable hypersurface in M whose first mean curvature is constant. We suppose that M admits a one-parameter group of conformal transformations such that the inner product α of the generating vector v^h and the normal C^h to the hypersurface does not change the sign (and $\neq 0$) on S. Then every point of S is umbilical.*

The main purpose of the present paper is to derive three integral formulas which are valid in a general Riemannian manifold and to generalise Katsurada's theorem to the case of general Riemannian manifolds admitting a one-parameter group of homothetic transformations.

§ 0. Preliminaries.

We consider an orientable $(m+1)$-dimensional Riemannian manifold M with positive definite metric and denote by g_{ji}, ∇_j, $K_{kji}{}^h$, $K_{ji}=K_{kji}{}^k$, the fundamental metric tensor, the covariant differentiation with respect to the Riemannian connection, the curvature tensor, and the Ricci tensor of M respectively, where and in the sequel the indices $h, i, j, k, \cdots$ run over the range $1, 2, \cdots, m, m+1$.

We assume that there is given an orientable hypersurface S whose local expression is

$$\xi^h=\xi^h(\eta^a)\,, \tag{0.1}$$

where ξ^h are local coordinates in M and η^a are local parameters on the hypersurface S, where and in the sequel the indices $a, b, c, d, \cdots$ run over the range $\dot{1}, \dot{2}, \cdots, \dot{m}$.