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A duality theorem for the real unimodular group of second order

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Introduction.

Let G be the real special linear group of second order. G consists of all real matrices, such that,

$$g=\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad ad-bc=1 .$$

The purpose of the present paper is to characterize G as a "dual group" of the space of its irreducible unitary representations. This space is given a law according to which the Kronecker product of any two representations is decomposed into irreducible components. This duality may be considered as an analogue of the Tannaka duality theorem for compact groups and of Pontrjagin duality theorem for locally compact abelian groups.

If A is a locally compact abelian group, there is the duality of Pontrjagin [1]. This duality is as follows. Any irreducible unitary representation of A is one-dimensional, i.e., a homomorphism of A into one-dimensional torus. This homomorphism is called a unitary character of A. In this case the Kronecker product of two irreducible representations is nothing but the ordinary product of characters as functions. Denote by $\hat{A}$ the totality of unitary characters χ, then $\hat{A}$ is an abelian group with the mentioned product. Moreover introducing the topology defined by uniform convergence on any compact set, $\hat{A}$ becomes a locally compact group.

We consider $\hat{\hat{A}}$ the group of the unitary characters on $\hat{A}$ in the same way, that is, an element of $\hat{\hat{A}}$ is a function $\tilde{\chi}(\chi)$ on $\hat{A}$, whose absolute value is unity. And we have

$$\tilde{\chi}(\chi_1)\tilde{\chi}(\chi_2)=\tilde{\chi}(\chi_1\chi_2)\,, \qquad \chi_1, \chi_2 \in \hat{A}\,.$$

This implies that $\tilde{\chi}$ commutes with the operation of product in $\hat{A}$, that is, the operation of Kronecker product of representations of A. When an element a of A is fixed, clearly the equality

$$\tilde{a}(\chi)=\chi(a)$$