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On Hartogs-Osgood's theorem for Stein spaces

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§ 1. Introduction.

1. In this paper, we shall show the following fact:

Let X be a connected normal Stein space with $\dim X \geqq 2$, *K a compact subset and D an open subset of X containing K. Assume that $D-K$ is connected. Then we have the followings:* (1) *Every holomorphic function in $D-K$ can be continued holomorphically into the whole D.* (2) *Every meromorphic function in $D-K$ can be continued meromorphically into the whole D.* (3) *Every holomorphic mapping of $D-K$ into a Stein space Y can be extended to that of D into Y.* (§ 5, Theorem 3).

When X is the complex Euclidean space C^n, (1) is well-known as a Hartogs-Osgood's theorem ([**10**], [**14**]) and (2) is essentially due to E. E. Levi ([**13**]), which were proved completely by A. B. Brown ([**4**]). In the previous paper H. Fujimoto-K. Kasahara [**7**], (1) for complex manifolds was discussed. For example, (1) is true if X is a Stein manifold.

The proof of the above, which we shall conclude in § 5, is divided into two parts. A method of the global continuation (§ 2, Theorem 1) is almost similar to that given in [**7**]. In the local continuation, a Bishop's theorem ([**3**]), from which we have easily a characterization of connected normal Stein spaces (§ 3, Theorem 2), plays the essential role. In § 4, we shall discuss some properties of real analytic functions, which will be used in § 5.

In the above fact, if we take off the assumption of the connectedness of $D-K$, we have the followings: (1′) For any holomorphic function f in $D-K$, we can find a holomorphic function in D which coincides with f on a non-empty open subset of $D-K$. (2′) and (3′) are (2) and (3) modificated in the same way, respectively. The proofs are included in Theorem 1 and Lemma 4.

2. For a Stein space X which is not normal, the homological codimension of X is related to the problem. (Cf. A. Andreotti-H. Grauert [**1**].) Our main theorem of this paper is the following:

THEOREM. *Let X be a Stein space with* $\operatorname{dih} X$ (*$=$the homological codimension of X*)$\geqq 2$, *K a compact subset and D an open subset of X containing K. Assume that each irreducible component of D is also irreducible in $D-K$. Then*