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Correction to "Some aspects of real-analytic manifolds and differentiable manifolds"

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In [2; Appendix, p. 139] we stated an extension theorem of C^s-mappings which was necessary to the proof of Classification Theorem of C^s-fibre bundles $(1 \leqq s \leqq \omega)$. However, the proof given there was incorrect and our reference to [3] was not pertinent to this theorem. Now we give a proof of the following extension theorem which corrects the theorem stated in [2, p. 139].

THEOREM. *Let M and N be C^ω-manifolds, and let L be a closed C^ω-submanifold of M. Suppose that we have a C^ω-mapping φ of L into N such that φ can be extended to a C^0-mapping f of M into N. Then, for any positive family $\mathcal{E}$, there exists a C^ω-mapping ϕ from M into N having the following properties:*

(i) *ϕ gives an $\mathcal{E}$-approximation to f in order* 0.

(ii) $\phi(p) \mid L = \varphi(p)$.

Here we formulate only real-analytic case, because differentiable case is trivial [cf. 2, p. 140]. If this theorem is established, then Theorem C [2, p. 138] remains valid.

In order to prove this theorem, we need two results due to H. Cartan [1].

PROPOSITION 1. *Any C^ω-function on L can be extended to a C^ω-function on M.*

PROPOSITION 2. *L can be defined as the zero points of a non-negative C^ω-function $\mu(p)$ on $M: L=\mu^{-1}(0)$.*

Proposition 2 is usually stated that L is defined as the common zero points of a finite number of C^ω-functions μ_i on M. Then we note that $\mu=\sum \mu_i^2$ satisfies the requirements of Proposition 2.

PROOF OF THEOREM. Imbed N in a Euclidean space E^k as a closed C^ω-submanifold. Then the given map φ of L into N can be written in coordinate components of $E^k: \varphi(p)=(\varphi^1(p), \cdots, \varphi^k(p))$. Applying Proposition 1 to each $\varphi^i(p)$, we get a C^ω-mapping Φ of M into E^k such that $\Phi(p)=(\Phi^1(p), \cdots, \Phi^k(p))$, and that $\Phi(p) \mid L=\varphi(p)$. We approximate f closely by a C^ω-mapping $\Psi(p)$ of M into N. Observe that $\Phi(p)$ and $\Psi(p)$ lie near each other in E^k when p is near L. Take a small neighborhood V of L and consider a C^ω-function $1/\mu(p)$ on V^c where $\mu(p)$ is a C^ω-function stated in Proposition 2. We extend