J. Math. Soc. Japan
Vol. 17, No. 2, 1965

An example for the theorem of W. Browder

By Seiya SASAO

(Received August 22, 1964)

Introduction

W. Browder proved in his paper [2] that a simply connected finite CW-complex of dimension $4k$ ($k \neq 1$) has the same homotopy type as a closed differentiable manifold[1] under the following conditions:

(1) Poincaré duality holds,

(2) there exists an oriented vector bundle ξ such that $T(\xi)$, the Thom space, has a spherical fundamental class,

(3) the Hirzebruch formula in the dual Pontrjagin classes of ξ gives the index.

In this paper we shall apply the above theorem to obtain the homotopy type classification of closed differentiable manifolds M which are simply connected and have homology groups $H^0(M) = H^4(M) = H^8(M) = Z$, $H^i(M) = 0$ $i \neq 0, 4, 8$. This result is previously obtained by J. Eells and N. Kuiper in [3]. Their method makes use of the existence of certain non-degenerate functions so that it is quite different from our method. They also obtained some informations on Pontrjagin classes, for instance a counter example of homotopy type invariance of Pontrjagin numbers, and examples of closed differentiable manifolds which have the same homotopy type but are not diffeomorphic. These results can be proved more intuitively by our method. Moreover, we shall give a counter example to the problem (2) about combinatorial and differentiable structures on manifolds proposed by C. T. C. Wall in A. M. S. Summer Topology Institute, Seattle, 1963, [4].

Let X_f be a CW-complex $S^4 \underset{f}{\cup} e^8$. If $h: S^7 \to S^4$ is the Hopf fibering X_h is the quaternion projective plane. Now we fix the orientation of S^4 and determine the orientation of (E^8, S^7) such that the generator of $H^8(E^8, S^7)$ represented by (E^8, S^7) is equal to $\bar{h}^* j^{-1}(e_h^4 \cup e_h^4)$ where $\bar{h}: (E^8, S^7) \to (X_h, S^4)$ is the characteristic map of the cell e^8, j is the inclusion homomorphism $H^8(X_h, S^4) \to H^8(X_h)$ and e_h^4 is the generator of $H^4(X_h)$ represented by the oriented S^4. Let (f) denote the homotopy class of a map $f: S^7 \to S^4$.

Since $\pi_7(S^4)$ is the direct sum $\boldsymbol{Z}(h) + \boldsymbol{Z}_{12}(\tau)$ where $2(h) + (\tau) = [i_4, i_4]$ we have

1) "closed" means compact and unbounded.