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Vector-valued holomorphic functions on a complex space

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§ 1. Introduction.

1. For a (reduced) complex space X and a Fréchet space F, an F-holomorphic function on X is defined to be an F-valued continuous function f on X if, for each continuous linear functional u on F, uf is holomorphic on X. In this paper, we attempt to extend some results in the theory of holomorphic functions of several complex variables to the case of F-holomorphic functions on X.

In [2] Bishop gave an expansion theorem, which asserts every F-holomorphic function f on a complex manifold is represented as a sum of (essentially) scalar-valued holomorphic functions and enables us to reduce the study of f to that of a sequence of ordinary holomorphic functions. Firstly, we generalize his expansion theorem to the case of F-holomorphic functions on a complex space. And, using this, we show an F-holomorphic function on a complex space is locally equal to the restriction of an F-holomorphic function in the ambiant space. Moreover, we get some theorems on the continuations and approximations of F-holomorphic functions, which include the following results:

(1) Let X' be a complex subspace of a complex space X. If each holomorphic function on X' is the restriction of a holomorphic function on X, then each F-holomorphic function on X' is also the restriction of an F-holomorphic function on X.

(2) Let X' be a subdomain of a complex space X. If (X, X') is a Runge pair, that is, each holomorphic function on X is compactly approximated on X' by holomorphic functions on X, then each F-holomorphic function on X' is also compactly approximated by F-holomorphic functions on X (§ 2).

2. Bishop introduced the notion of the vectorization S_F of a coherent analytic sheaf S with respect to a Fréchet space F and gave some interesting properties of it ([2]). These are proved essentially by his expansion theorem. Using our generalized expansion theorem for F-holomorphic functions on a complex space, we can generalize almost all results of Bishop [2] to the case