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On some properties of a proximity

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§ 1. Introduction.

Efremovich [1] defined a relation δ on a set, called a *proximity*. For a pair of subsets A and B of a point set R we usually write $A\delta B$ if A and B are proximate, otherwise $A\bar{\delta}B$. Throughout this paper we shall use the notations $(A, B)\in\delta$ and $(A, B)\notin\delta$ instead of $A\delta B$ and $A\bar{\delta}B$ respectively. (See Pervin [2].)

Efremovich required that the relation δ should satisfy the following four axioms:

Axiom 0. (Symmetry) $(A, B)\in\delta$ if and only if $(B, A)\in\delta$.

Axiom 1. Both $(A, C)\notin\delta$ and $(B, C)\notin\delta$ if and only if $(A\cup B, C)\notin\delta$.

Axiom 2. For arbitrary two points $a, b\in R$, $(\{a\}, \{b\})\in\delta$ if and only if $a=b$.

Axiom 3. (Separation) If $(A, B)\notin\delta$ then there are disjoint subsets U and V of R such that $(A, R-U)\notin\delta$ and $(B, R-V)\notin\delta$.

Efremovich [1, p. 196] showed that every proximity on a set R yields a completely regular space if one defines the topology of R as follows: a subset U of R is a neighborhood of $A\subset R$ if and only if $(A, R-U)\notin\delta$. This definition can be replaced by the following: (♯) a subset G of R is defined to be open if and only if $(\{x\}, R-G)\notin\delta$ for every $x\in G$. (See Császár et Mrówka [3, p. 195].)

In this paper we shall first (§ 2) define slightly different axioms from Efremovich's. In § 3 we shall show that our proximity on a set yields a completely normal space. The last section 4 will be devoted to an example of our proximity on a set.

§ 2. Definitions and lemmas.

By a *paraproximity* on a set R we mean a relation δ for pairs of subsets of R satisfying the following axioms:

Axiom I. *$(A, \phi)\notin\delta$ for every $A\subset R$.*