J. Math. Soc. Japan
Vol. 16, No. 4, 1964

The kernel representation of the fractional power of the strongly elliptic operator

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(Received Feb. 4, 1964)
(Revised July 1, 1964)

Introduction.

Let A be a strongly elliptic partial differential operator of order $2m$ defined in a domain D of R^n, and let us consider the Dirichlet problem for the operator $A+\lambda I$, λ being a complex number. Then we can define the fractional power $A^{-\alpha}$ under a suitable condition on the spectrum of A. This operator is continuous from $L^2(D)$ into itself, if $\mathrm{Re}\,\alpha>0$. In the case where A is formally self-adjoint, T. Kotake and M. S. Narasimhan [2] have recently proved that $A^{-\alpha}(\mathrm{Re}\,\alpha>0)$ has the kernel representation and moreover this kernel is very regular. In this article, we want to prove the same result for not necessarily self-adjoint operator.

In § 1, we summerize some well-known facts on the Green operator attached to the Dirichlet problem in the space $L^2(D)$, and impose a condition (C) on the spectrum of A. In § 2, we express weak solutions $u\in L^2(D)$ of the equation $Au+\lambda u=f\in L^2(D)$ by means of a parametrix (formula (2.7) below), and we also express the Green kernel $K(\xi, x\,|\,\lambda)$ of the operator $A+\lambda I$ by using both the parametrix and the Green operator G_λ ((2.13)). We should mention here that these expressions have been obtained by H. G. Garnir in the case of metaharmonic functions [1] and it has played a fundamental rôle in the study of the Green kernel. In § 2 and § 3, we assumed the existence of such a parametrix $E(x, \xi\,|\,\lambda)$ with certain properties. The existence of such a parametrix will be proved in § 4.

The author wishes to express his thanks for Professor Mizohata, who suggested the utilization of parametrix on this subject.

§ 1. Green operator G_λ.

We deal with the strongly elliptic partial differential operator of order $2m$ defined in a domain D (bounded or unbounded) of R^n