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Martin boundary for linear elliptic differential operators of second order in a manifold

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§1. **Introduction.** The generalized boundary value problem of Dirichlet type for the Laplace operator in arbitrary region D of Euclidean space R^N was studied by R. S. Martin [8]. It is shown in [8] that any non-negative harmonic function on D is represented in the form of an integral over the set of so-called minimal positive harmonic functions; the set of minimal functions corresponds to 'ideal boundary' of D which we call Martin boundary. On this subject, we may find in [2] work of a more general nature.

Recently the theory of Martin boundary has been connected with the theory of Markov processes; Martin boundaries for Markov chains have been constructed by many authors, especially by J. L. Doob [3], G. A. Hunt [4] and T. Watanabe [10]. M. G. Šur [9] has constructed the Martin boundary for the linear elliptic operator of second order in an arbitrary region $D \subset R^N$, which corresponds to a diffusion process in D. His method is achieved along the contents of Martin's paper [8], but mostly due to the probabilistic treatment.

In the present paper, we shall construct the Martin boundary for a linear elliptic differential operator of second order in a subdomain D of a manifold $\boldsymbol{M}$ (D may coincide with $\boldsymbol{M}$) by means of purely analytical treatment, as a direct extension of Martin's method, and show that, if a part S of the boundary ∂D of the domain D considered in $\boldsymbol{M}$ is 'smooth' and the elliptic operator is regular on $D+S$, then S is homeomorphically imbedded into the Martin boundary; in general, we do not assume any regularity of ∂D and any restriction on the behavior of the elliptic operator near ∂D. Our method is essentially same as Martin's method in [8] except the result on the correspondence between the smooth part of ∂D and a subset of the Martin boundary. However we use some properties of fundamental solutions of diffusion equations shown in the author's previous papers [5], [6], instead of some classical results in potential theory which are well known in the case of usual Laplace operator but whose extension to the case of general elliptic operators is not necessarily evident.

The contents of the present paper are as follows. In §2, we state some