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4-connected differentiable 11-manifolds with certain homotopy types

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Introduction.

J. Milnor [8] and S. Smale [11] have proved that the oriented differentiable homotopy $(4k-1)$-spheres $(k>1)$ (i.e. $(4k-1)$-manifolds which have the homotopy type of the $(4k-1)$-sphere), which are boundaries of π-manifolds, are homeomorphic to the natural sphere S^{4k-1} and their diffeomorphism classes form a cyclic group $\Theta^{4k-1}(\partial\pi)$ of a finite order under the connected sum operation. It is known (cf. [8]) that in general any 7 or 11 dimensional closed (i.e. compact unbounded) oriented differentiable π-manifold always bounds a π-manifold. Thus the group Θ^7 (resp. Θ^{11}) of diffeomorphism classes of oriented differentiable homotopy 7-spheres (resp. 11-spheres) coincides with $\Theta^7(\partial\pi)$ (resp. $\Theta^{11}(\partial\pi)$) and hence homotopy 7-spheres (resp. 11-spheres) have been completely classified diffeomorphically as oriented manifolds. So it has turned out that there exist precisely 28 (resp. 992) distinct diffeomorphism classes of homotopy 7-spheres (resp. 11-spheres). (In the following we shall express this situation by saying: there exist precisely 28 (resp. 992) distinct differentiable manifolds on homotopy 7-spheres (resp. 11-spheres).)

In this paper we shall consider $(2k-2)$-connected closed oriented differentiable $(4k-1)$-manifolds which bound π-manifolds and whose $(2k-1)$-th homology groups are cyclic groups of orders n which are products of distinct prime numbers. They are all boundaries of so-called handlebodies (S. Smale [11], [12]). We shall denote the set of such manifolds with $\partial\mathcal{H}'_n(2k)$. We shall see that the homotopy type of such manifolds is uniquely determined by k and n, and shall be able to determine the numbers of differentiable manifolds of such homotopy types, when $n=p$ (a prime number).

I. Tamura [17] has proved that there exist precisely 56 differentiable 7-manifolds of the homotopy type of manifolds of $\partial\mathcal{H}'_3(4)$ and that they are obtained from the standard one by forming connected sums with elements of Θ^7 and the orientation-reversing. In the following we shall show that there exist precisely 1984 differentiable 4-connected 11-manifolds of the homotopy type of manifolds of $\partial\mathcal{H}'_p(6)$ for each prime p (resp. precisely 56 differentiable 2-connected π-manifolds of dimension 7 of the homotopy type of manifolds of