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Some aspects of real-analytic manifolds and differentiable manifolds

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Introduction

In 1958 C. B. Morrey [7] and H. Grauert [3] proved that any real-analytic manifold can be real-analytically imbedded in a Euclidean space by a regular and proper mapping. This, combined with the result of H. Whitney [12], shows that any differentiable manifold has a unique real-analytic structure, or in other words, every manifold has as many C^{ω}-structures as C^1-structures. We shall refer to this fact in speaking of the "constancy of differentiable structure" of manifolds.

Now, a fundamental tool in Whitney's work [12] was the approximation theorem, saying that any differentiable mapping f between two real-analytic manifolds M, N can be arbitrarily well approximated by a real-analytic mapping φ (cf. § 1 for an exact formulation). Actually Whitney [12] proved this under the condition that M and N are realized in a Euclidean space, but this condition can be removed owing to the result of Morrey-Grauert [7], [3]. From this it follows in particular that to any regular mapping f we can find a regular real-analytic approximation φ. Thus φ will be an analytic homeomorphism if f is a homeomorphism; i. e., the uniqueness of C^{ω}-structure compatible with a C^1-structure of a manifold—one half of the "constancy of differentiable structure"; another half being the existence of C^{ω}-structure—is a direct consequence of the approximation theorem.

In the present paper, we shall first state the generalized approximation theorem and some immediate consequences of it (§ 1). Now, corresponding to the case where f is injective, the approximation theorem has an application to fibre spaces; any differentiable fibre space $P=P(B, \pi)$ possesses a unique real-analytic structure as a fibre space when the projection π is proper (§ 2). On the other hand, in the case where f is injective, the approximation theorem combined with G. D. Mostow's theorem [8] concerning the equivariant imbeddings yields results related to transformation groups, one of which is formulated as follows: Let G be a compact Lie group acting on a compact real-analytic manifold M as a C^1-transformation group. Then G necessarily acts