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On the sample paths of the symmetric stable processes in spaces

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§ 1. Introduction and Summary.

Let $\{X(t); t \geqq 0\}$ be the symmetric stable process in R^N of index α with $0 < \alpha \leqq 2$; that is, a stochastic process with stationary independent increments such that continuous transition density, $f(t, x-y)$, relative to the Lebesgue measure in R^N is uniquely determined by its Fourier transform

$$e^{-t|\xi|^{\alpha}} = \int_{R^N} e^{i(x,\xi)} f(t, x) dx .$$

Here ξ and x are points in R^N, dx is the N-dimensional Lebesgue measure. The notation (x, ξ) means the usual inner product in R^N and $|x| = (x, x)^{1/2}$ is the usual Euclidean norm. When $\alpha = 2$, $X(t/2)$ is the standard Brownian motion process. It is well-known that we may assume that the sample functions of X have certain regularity properties. To be precise we may assume that almost all sample functions have right continuity and left-hand limits everywhere and are bounded on each bounded parameter set. We will write P_x and E_x for the conditional probability and expectation under the condition $X(0) = x$. Unless otherwise stated we assume $X(0) = 0$ with probability one, and we use the abbreviations $P_0 = P$ and $E_0 = E$. Our process is defined over some basic probability space $(W, \mathfrak{B}, P)$. We will often suppress the w's in our notation.

The main purpose of the present paper is to discuss some properties of the path functions in which the dimension number N plays an essential rôle. From this point of view, the pioneers were G. Pólya and S. Kakutani. In case of the stable process, such properties vary also according to the index α and were investigated by H. P. McKean [**12**] and others. For example, our process is recurrent if $N \leqq \alpha$, while if $N > \alpha$ then it is non-recurrent.

The first problem is concerned with the speed of wandering off to infinity in the transient case, that is, the problem of giving a characterization of upper or lower classes of monotone decreasing functions which limit the speed. It is an extension of the result by Dvoretzky-Erdös [**6**]. Our method is based