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Differential forms of the first and second kind on modular algebraic varieties

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Let V be a complete non-singular variety. We denote the universal domain by K and its characteristic by p. All vector spaces and their dimensions will mean those with respect to K. Differential forms of the first kind of degree r on V are, as is well known, elements of $H^0(V, \Omega^r)$ where Ω^r is the sheaf of germs of holomorphic differential forms of degree r. The dimension of this space is denoted by $h^{r,0}$. We define the differential forms of the second kind on V after Picard and Rosenlicht [**9**] as follows. Let ω be a differential form of degree $r \geqq 1$ on V; we call ω to be *of the second kind at a point* P of V if there exists a differential form θ_P of degree $r-1$ on V such that $\omega - d\theta_P$ is holomorphic at P; if ω is of the second kind at every point of V, we call ω to be *of the second kind on* V. We denote by $\mathcal{D}_2^{(r)}(V)$ the space of all closed differential forms of the second kind of degree r on V, and by $\mathcal{D}_e^{(r)}(V)$ that of all exact differential forms among them. The dimension of the factor space $\mathcal{D}_2^{(1)}(V)/\mathcal{D}_e^{(1)}(V)$ is known to be: (1) $2h^{1,0}$ or $h^{1,0}$ respectively, in case $\dim V = 1$, according as $p = 0$ or > 1 (Rosenlicht [**9**]), (2) $2h^{1,0}$ in case $K = \boldsymbol{C}$, $\dim V$ being arbitrary (Hodge-Atiyah [**5**]).

Our purpose in § 1 is to show that the dimension of the factor space $\mathcal{D}_2^{(1)}(V)/\mathcal{D}_e^{(1)}(V)$ is $h^{1,0}$, whenever $p > 1$, $\dim V$ being arbitrary. We shall prove this in making use of the operator C of Cartier; a proof of this fact in case $\dim V = 1$, making also use of C, has been given by Cartier (Tamagawa's lecture in Tokyo University 1960) and Barsotti [**2**, p. 63], but even in this case our proof is based on other property of C than that used by them. Theorem 2 generalizes Theorem 2 of [**9**]. In § 2, we shall give some results on closed semi-invariant differential forms on modular abelian varieties; in case of characteristic zero, the corresponding results are found in Barsotti [**1**]. In § 3 we shall prove the following result. It is known that the dimension q of the Albanese variety of a variety V is $\leqq h^{1,0}$ (Igusa [**6**]); there is a famous example of V due to Igusa [**7**] for which the strict inequality $q < h^{1,0}$ holds. We shall prove that if V is defined over a field of prime characteristic p and if V has p-torsion divisors, then the inequality $q < h^{1,0}$ holds.