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Note on holomorphically convex complex spaces

By Joji KAJIWARA

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Hirzebruch [4] proved that for any 2-dimensional complex space Y there exists a 2-dimensional complex manifold X which is obtained by a proper modification of Y in the inuniformisable points of Y. If Y is a Stein space, then X is obviously a holomorphically convex complex manifold. In the present paper we shall conversely consider the conditions that a holomorphically convex complex space can be obtained by a proper modification of a Stein space. (In the present paper we mean by a complex space an α space $=\beta_n$ space in Grauert-Remmert [3].)

The following lemma is a special case of the theorem of factorization of Remmert-Stein [9].

LEMMA 1. *Let ζ be a proper holomorphic mapping of an n-dimensional connected complex space X onto an n-dimensional Stein space Y such that ζ induces an isomorphism of the integral domain $I(Y)$ of all holomorphic functions in Y onto the integral domain $I(X)$ in X. Then (X, ζ, Y) is a proper modification. Moreover, if each connected component of the set of degeneracy E of ζ is compact in X, then (X, ζ, Y) is a proper points-modification.*

PROOF. If $n=1$, ζ is biholomorphic. Therefore we may assume that $n \geqq 2$. Let x be any point of X. We denote by σ_x the connected component of $\zeta^{-1}\zeta(x)$ containing x. σ_x is a nowhere discrete connected compact analytic set in X if $x \in E$. We shall introduce an equivalence relation R in X as follows;

$$x \text{ and } y \in X \text{ are equivalent modulo } R \text{ if } \sigma_x = \sigma_y .$$

Let $X^* = X/R$ be the factor space of X by the equivalence relation R. If we consider the canonical mappings $\lambda : X \to X^*$ and $\zeta^* : X^* \to Y$, then the commutativity holds in the following diagram;