J. Math. Soc. Japan
Vol. 15, No. 3, 1963

Note on the computation of Bessel functions through recurrence formula

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(Received May 9, 1963)

§1. Introduction. The problem.

As is well-known, Bessel function of the first kind $J_n(x)$ satisfies the recurrence formula

$$J_{n+1}(x)-\frac{2n}{x}J_n(x)+J_{n-1}(x)=0\,. \tag{1}$$

For a fixed value x, we may compute the values of $J_2(x), J_3(x), \cdots$ through the formula (1), if we know the values of $J_0(x)$ and $J_1(x)$. However, this method does not fit to the practice, because the formula (1) implies serious *unstability*[1]. Even if the initial values of $J_0(x)$ and $J_1(x)$ have a little error, the successive values of $J_n(x)$ given by (1) will, in general, tend to $+\infty$ or $-\infty$, although the true values of $J_n(x)$ must tend to 0 when $n\to+\infty$.

On the other hand, it is useful to apply the formula (1) from large values of n to the smaller ones. Precisely speaking, the value of $J_n(x)$ is computed by the following algorism.

1°. Choose sufficiently large N, which will be discussed later, and put

$$j_{N+1}^*=0, \qquad j_N^*=\varepsilon\,.$$

Here ε is usually taken as the smallest positive number admissible in the computor, viz., 10^{-10} or 2^{-128}, etc.

2°. Compute j_n^* $(n=N-1, N-2, \cdots, 1, 0)$ by the recurrence formula

$$j_{n-1}^*=\frac{2n}{x}j_n^*-j_{n+1}^*\,. \tag{1'}$$

3°. Noting the relation

$$J_0(x)+2\sum_{n=1}^{\infty}J_{2n}(x)=1\,,$$

the values of $J_n(x)$ are obtained by

$$J_n(x)=\frac{1}{K}j_n^* \tag{2}$$

where we have put

1) This unstability is well known; e.g. see Uno [4].