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On the field of moduli of an abelian variety with complex multiplication

Dedicated to Professor Y. Akizuki on his sixtieth birthday

by Koji DOI

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The theory of complex multiplication of abelian varieties has been established by A. Weil, G. Shimura, and Y. Taniyama ([3], to be referred as [CM]). The main parts of them, i. e. the so-called construction of class-fields are concerned with simple abelian varieties (primitive CM-type). Naturally as the next step the extension of the theory to the case of composite varieties should be considered.

Along this line we shall begin with a very simple case, assuming the variety to be a direct product $B_1 \times \cdots \times B_h$ of simple abelian varieties B_i of the same CM-type. The result of this note consists in Main Theorem of §4 and Corollary to it. Our result implies the

THEOREM. *The field of moduli of the product* $B_1 \times \cdots \times B_h$ *(with respect to any polarization) is contained in the class-field obtained from the field of moduli of the factor* B.

This means that we can not get any different class-field to that of primitive CM-type from such product.

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We shall use the same notations and terminologies as in [CM].

§1. Let $(K; \{\varphi_i\})$ be a primitive CM-type and $[K; \boldsymbol{Q}] = 2n$. We shall consider a couple (A, θ) formed by an abelian variety A defined over $\boldsymbol{C}$ and an isomorphism θ of K into $\mathcal{A}_0(A)$. Let h be a positive integer. We say that (A, θ) is of type $(K; \{\varphi_i\}; h)$ if the following two conditions are satisfied.

(A 1) $\dim A = nh$.

(A 2) For any element $\alpha \in K$, the analytic representation of $\theta(\alpha)$ (cf. [CM; §3.2.]) is equivalent to the diagonal matrix whose diagonal elements are exactly h times $\alpha^{\varphi_1}, \cdots, \alpha^{\varphi_n}$. If (A, θ) is of type $(K; \{\varphi_i\}; 1)$ then (A, θ) is of type $(K; \{\varphi_i\})$ in the sense of [CM, §5.2]. Therefore we put $(K; \{\varphi_i\}; 1) = (K; \{\varphi_i\})$.

PROPOSITION 1. *If* (A, θ) *is of type* $(K; \{\varphi_i\}; h)$, *then* A *is isogenous to a*