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# Martin boundaries for certain Markov chains

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## 1. Introduction.

We shall be concerned with the theory of Martin's ideal boundary as adapted for Markov chains by J. L. Doob in [1]. This theory has been developed by Hunt in [4] and by Watanabe in [8]; some applications and illustrations are contained in works by Doob, Snell and Williamson [2], Dynkin and Malyutov [3], and Watanabe [9]. However, the specific cases to which the boundary theory has been applied always fall under the general heading of homogeneous processes (usually sums of independent lattice random variables). In the present paper we shall give some theorems describing and applying the Martin boundary theory associated with certain other classes of Markov chains. We assume the reader is familiar with the basic ideas of the Martin boundary; [1], [4], [8] contain detailed treatments, and short sketches of the theory can be found in [2] and [9].

It seems to us that there are three purposes which a study of this type can serve. The first is that the elegance and interest of the general Martin theory makes it desirable to have a variety of specific examples; as noted above, those so far studied are all of one general kind. The second thing is that the specific results may be of some analytical interest. The central Martin representation theorem provides, in our examples, the general positive solution to certain partial difference equations. In addition to these solutions and some of their properties we obtain (in § 4) amusing characterizations of sequences analogous to moment sequences obtained from orthogonal polynomials. Finally, some results of primarily probabilistic interest are found; these include a 0—1 law (§ 2) and some limit theorems for certain Markov processes.

The Martin boundary for a denumerable Markov chain is defined in [1] relative to a preferred state which we denote by 0. We denote by $R$ the set of all states which can be reached from 0, and by $P$ the matrix which gives the transition probabilities between states of $R$. Given such a chain we shall also consider a second one called the space-time chain for $P$. A state of this new chain represents the position and the time in the previous chain; we