Local theory in function analysis*

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§ 0. Introduction

The aim of this paper is, as a continuation of the previous papers [10], [11], to establish the local theory in algebro-topological systems over commutative AW*-algebras.

This paper consists of two parts. In each part, we shall explain how to reduce properties of elements of algebro-topological systems over commutative B*-algebras to those of elements of (classical) algebro-topological systems over the field of complex (or real) numbers by making use of the local theory. In § 1, we shall establish a theorem concerning quasi-ordered linear spaces (Theorem A) and the extension theorem of H. Hahn and S. Banach (Theorem B). In § 2, we deal with a theorem of I. Gelfand (Theorem C) and a theorem of S. Mazur and I. Gelfand (Theorem D). These theorems will be discussed for the case of algebro-topological systems over commutative B*-algebras (for example, quasi-ordered linear spaces over commutative B*-algebras, linear spaces over commutative B*-algebras, Banach algebras over commutative B*-algebras and B*-algebras over commutative B*-algebras). They are, however, essentially valid for the case of algebro-topological systems over commutative AW*-algebras, which were originated by I. Kaplansky [6] and investigated by H. Widom [14] and M. Nakai [9]. Precisely speaking, we consider a compact Hausdorff space Ω and the commutative B*-algebras C(Ω) (or R(Ω)) of complex- (or real-) valued continuous functions defined on Ω. Suppose there is a theorem concerning an algebro-topological system over C(Ω) (or R(Ω)). Then we shall say that this theorem is, for instance, of Stonian class if it is valid for the case that the underlying space Ω is Stonian and if further there exists without fail a counter example, that is, an example, for which the theorem does not hold, provided that Ω is not Stonian. In this sense, these theorems are exactly of Stonian class. (L. Nachbin [8], D.B. Goodner [2], J.L. Kelley [7], M. Nakai [9], and M. Hasumi [3] proved that the extension theorem of H. Hahn and S. Banach is exactly of Stonian class.)

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