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On irreducibility of an analytic set

By Kenkiti KASAHARA

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§1. Let D be a domain in the n-dimensional complex Euclidean space $\boldsymbol{C}^n$, and M be a k-dimensinal analytic set[1] in D ($1 \leqq k \leqq n-1$). It is well-known that the set of all irreducible points of M is not always an open subset of M. For example, the analytic set $\{z_1^2 - z_2^2 z_3 = 0\}$ in $\boldsymbol{C}^3$ is irreducible at the origin, but there exist reducible points of the analytic set converging to the origin (Osgood [2]). We shall say that a point p is *a singular irreducible point of* M, if M is irreducible at p and there exist reducible points of M converging to p. *Let* S *be the set of all singular irreducible points of* M. Recently S. Hitotumatu [1] has shown that S must be empty if M is an analytic set of 1-dimension in $\boldsymbol{C}^2$. In this note, we show the following:

THEOREM. *The closure* $\bar{S}$ *of* S *in* D *is an analytic set in* D. *For each point* $p \in \bar{S}$, *a relation* $\dim_p \bar{S} \leqq \dim_p M - 2$ *holds.*

REMARK. For the set S itself, Theorem is not true. For example, the analytic set $\{z_4(z_1^2 - z_2^2 z_3) = 0\}$ in $\boldsymbol{C}^4$ has the set $\{z_1 = z_2 = z_3 = 0,\ z_4 \neq 0\}$ as S. For another example, the analytic set $\{z_4^4 - 2z_3^2 z_4^2 + z_3^4(1 - z_1^2 z_2) = 0\}$ in $\boldsymbol{C}^4$ is irreducible in $\boldsymbol{C}^4$. Outside the set $\{z_2 = 0\} \cup \{z_3 = 0\} \cup \{1 - z_1^2 z_2 = 0\}$, the analytic set is decomposed into the following four sets:

$$\{z_4 = z_3\sqrt{1 + z_1\sqrt{z_2}}\}, \qquad \{z_4 = -z_3\sqrt{1 + z_1\sqrt{z_2}}\},$$

$$\{z_4 = z_3\sqrt{1 - z_1\sqrt{z_2}}\} \quad \text{and} \quad \{z_4 = -z_3\sqrt{1 - z_1\sqrt{z_2}}\}.$$

We have easily

$$S = \{z_1 = z_2 = 0,\ z_3 = z_4\} \cup \{z_1 = z_2 = 0,\ z_3 = -z_4\} - \{(0, 0, 0, 0)\}.$$

But we can generally show that the set S itself has an analytic property, that is, S is locally the finite union of locally analytic sets. (cf. §4.)

First applying the Remmert-Stein's 'Einbettungssatz' ([3]) and the method of Osgood [2, Chap. II, §15], we shall define the number of components of M at a point $p \in M$. (cf. §2). In §3, we shall derive a property of roots of a polynomial. In §4, we shall consider Theorem for the case that M is

1) About the definition and related notions of an analytic set, see Remmert-Stein [4].