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On the pseudo-conformal geometry of hypersurfaces of the space of n complex variables

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Introduction

By a hypersurface we here mean a $(2n-1)$-dimensional real analytic submanifold of the space of n complex variables, i. e. the n-dimensional complex Cartesian space $\boldsymbol{C}^n (n \geqq 2)$. A homeomorphism f of one hypersurface S onto another hypersurface S' is called a pseudo-conformal homeomorphism, if it can be extended to a complex analytic homeomorphism of a neighborhood of S onto a neighborhood of S' (Definition 1). In case such f exists, we say that the two hypersurfaces S and S' are mutually pseudo-conformally equivalent.

The main purpose of this paper is to study conditions for the pseudo-conformal equivalence of two hypersurfaces. In case $n=2$, this problem was first considered by H. Poincaré and was studied by B. Segre and E. Cartan. In his paper [**1**], E. Cartan gives a complete solution of the problem by the application of his own "method of the equivalence" [**3**]. We want to generalize his results to case $n \geqq 2$.

We introduce the notion of a non-degenerate hypersurface (Definition 2) which is a slight generalization of the notion of a hypersurface satisfying the so-called condition of Levi-Krzoska. Moreover, we introduce the notion of a regular hypersurface (Definition 3). Roughly speaking, a non-degenerate hypersurface is regular when it locally admits a non-trivial infinitesimal pseudo-conformal transformation (Proposition 5). Now, the main theorem (Theorem 4) in this paper may be stated as follows: To every regular non-degenerate hypersurface S there is associated, in an intrinsic manner, a principal fiber bundle P over the base space S together with an infinitesimal structure B in P, in terms of which the pseudo-conformal equivalence (of two regular non-degenerate hypersurfaces) can be characterized. The infinitesimal structure B stated above is a Cartan connection which we shall call the normal pseudo-conformal connection associated to the hypersurface S, cf. [**2**]. One finds that the situation is just analogous to the case of the Riemannian geometry of hypersurfaces. As an application of Theorem 4, it is shown that if a hypersurface S has a non-degenerate part, then the group $G(S)$ of all the pseudo-conformal transformations of S becomes a Lie group of dimension