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Dimension-theoretical structure of locally compact groups

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This paper is devoted to the study of dimension-theoretical structure of locally compact groups and their factor spaces. Montgomery-Zippin [9] proved that every finite-dimensional, locally compact group is a generalized Lie group and finally Yamabe [17] proved that every locally compact group is also a generalized Lie group. These are the most important results not only for the group-theoretical structure of locally compact groups but also for the dimension-theoretical structure of such groups. Montgomery [7] had proved also, before his fundamental theorem cited above was established, that the invariance theorem of a domain is true in finite-dimensional, locally connected, locally compact, separable metric groups. P. Alexandroff conjectured that the covering dimension of any locally compact group coincides with its inductive dimension. Recently this conjecture has been solved in the affirmative by Pasynkov [15]. His result will be generalized in § 2, after some preliminaries of § 1, for factor spaces of finite-dimensional locally compact groups by connected compact subgroups. It will also be proved that $\dim G=\dim H+\dim G/H$, where dim denotes the covering dimension, for any locally compact group G and any closed subgroup H of it. Montgomery-Zippin [8], Yamanoshita [18] and others have considered the dimension of factor spaces of locally compact groups and obtained the equality for some special cases. Our theorem seems to be a complete answer for the problem concerning the covering dimension of factor spaces of locally compact groups. In § 3 the decomposition theorem for locally compact groups will be proved. Both Pasynkov's theorem cited above and the author's decomposition theorem show that there are some analogy between the dimension-theoretical structure of locally compact groups and that of Euclidean spaces. In § 4 we shall point out a difference between the two by proving that the invariance theorem of a domain is not true in any finite-dimensional, locally compact, metric group which is not locally connected. Combining this with Montgomery's invariance theorem mentioned above, we know that a finite-dimensional, locally compact, metric group is locally connected (or equivalently a Lie group) if and only if the invariance theorem is valid in it.

In this paper a topological group means a T_1-group. Hence a locally compact group and its factor space by a closed subgroup are always normal