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Generalization of a theorem of Paley and Wiener

By Takao KAKITA[1)]

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1. Introduction

In his previous paper [2], the author has proved that a theorem of Planchrel and Polya [3], which contains the theorem of Paley and Wiener in one-dimensional form, can be extended to the case in which distribution is involved.

In this paper, we shall give an extension of Stein's theorem [5] in a completely general form by modifying his method.

Our aim is to show that all distributions whose Fourier transforms vanish outside a given compact symmetric and convex domain in n-space are characterized in a one-dimensional form. It is another generalization of the theorem of Paley and Wiener, through the removal of the imposed condition of boundedness on $f(x)$, which gives an extension of a theorem due to Stein [5]. It is my pleasure to thank Professor G. F. D. Duff for taking the trouble to read over this manuscript.

2. Stein's theorem

Adopting Stein's notations, we shall denote by E_n the euclidean n-space, and by $x=(x_1, \cdots, x_n)$ a generic point in it. E^n will denote the dual euclidean n-space by the inner product

$$x \cdot y = \sum_{i=1}^{n} x_i y_i .$$

The Fourier transform of $f(x) \in L^2(E_n)$ is given by

$$\mathcal{F}[f](y) = 1/(2\pi)^{\frac{n}{2}} \int_{E_n} e^{-ix\cdot y} f(x)\, dx .$$

Let Ω be a compact, convex and symmetric domain in E^n, and let $\Omega^* = \{x \in E_n ; |x \cdot y| \leqq 1 \text{ for all } y \in \Omega\}$. By $c(x)$ we mean the characteristic function of Ω^*. Define

$$U_t(f)(x) = f * c_t(x)$$

where $f(x)$ is a locally integrable function and $c_t(x) = t^{-n} c(x/t)$.

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