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Differentiable 7-manifolds with a certain homotopy type

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J. Milnor [**10**] has determined the so-called J-equivalence (h-cobordism) classes of oriented differentiable 7-manifolds having the homotopy type of the 7-sphere, and S. Smale [**13**] has proved that such manifolds are homeomorphic to the 7-sphere and the J-equivalence classes are the same as the diffeomorphic classes in this case. Thus compact unbounded oriented differentiable 7-manifolds which are homotopy spheres were completely determined. There exist precisely 28 such differentiable 7-manifolds which form a cyclic group Θ^7 under the connected sum.

In this note we shall consider compact unbounded 2-connected oriented differentiable 7-manifolds whose third homology groups are cyclic of order 3, having trivial Steenrod operations. We shall show that there exist precisely 56 differentiable 7-manifolds of this homotopy type and that they are obtained from the standard one by connected sums of elements of Θ^7 and the orientation-reversing.

1. Let M^7 be the compact unbounded 2-connected oriented (C^∞-) differentiable 7-manifold such that $H_3(M^7; Z) \approx Z_3$ and that the Steenrod operation $\mathcal{P}_3^1: H^3(M^7; Z_3) \rightarrow H^7(M^7; Z_3)$ is trivial, namely, for $u \in H^3(M^7; Z_3)$

$$(P) \qquad \mathcal{P}_3^1(u) = 0 .$$

LEMMA 1. *The condition* (P) *is equivalent to* $p_1(M^7)=0$, *where* $p_1(M^7)$ *is the first Pontrjagin class of* M^7.

PROOF. This lemma follows from the formula given by Hirzebruch [**6**]:

$$p_1(M^7) \cup u = \mathcal{P}_3^1(u) \qquad \bmod 3$$

for $u \in H^3(M^7; Z_3)$.

LEMMA 2. M^7 *is a* π*-manifold.*

PROOF. Suppose that M^7 is imbedded in a high dimensional Euclidean space R^{7+N}. Denote by ν^N the normal bundle of M^7. Let K be a triangulation of M^7. Let us define a (continuous) field of normal N-frames on M^7 by stepwise extensions on the skeletons $K^{(q)}$ $(q=0, 1, \cdots, 7)$ of K using the obstruction theory in the well-known manner. Since $H^q(M^7; Z)=0$ $(q=1, 2, 3)$ and $\pi_2(SO(N))=0$, we can define a field f of normal N-frames on $K^{(3)}$. Let $c(f) \in Z^4(M^7; Z)$ be the obstruction cocycle to extend f in $K^{(4)}$, Then the first