J. Math. Soc. Japan
Vol. 14, No. 3, 1962

# On groups of automorphisms of algebraic varieties

By Makoto ISHIDA

(Received Nov. 27, 1961)

Let $V$ be a complete, non-singular variety and let $G(V)$ be the group consisting of all the automorphisms of $V$. Then we can define two matrix representations $M^{(V)}$ and $S^{(V)}$ of $G(V)$. The representation $M^{(V)}$ is defined by means of the $l$-adic representation of the ring of endomorphisms of an Albanese variety attached to $V$ (with a fixed rational prime $l$ different from the characteristic of the universal domain) (cf. [1] and [4]). On the other hand, the representation $S^{(V)}$ is defined also by means of the matrix representation of linear transformations of the space of the linear differential forms of the first kind on $V$ (with respect to a fixed basis of it) (cf. [1] and [3]). While the field of coefficients of $M^{(V)}$ is always of characteristic zero, the field of coefficients of $S^{(V)}$ is contained in the universal domain under consideration and so some difficulties occur for the study of $S^{(V)}$ in the case of positive characteristics.

The purpose of this paper is to give some informations about these two representations $M^{(V)}$ and $S^{(V)}$ (or, rather, the restrictions of them to a finite subgroup of $G(V)$). Since our results are well-known when the characteristic of the universal domain is equal to zero (cf. the remark in the section 2), we shall restrict ourselves to the case of positive characteristics. First we consider the case where $V=A$ is an abelian variety, which is the case of importance as seen later. In particular, it is shown that $S^{(A)}$ gives a faithful representation of a finite multiplicative group consisting of endomorphisms of $A$, provided its order is prime to the characteristic of the universal domain. Secondly, we show a relation between two representations $M^{(A)}$ and $S^{(A)}$, which is suggested by a classical result. In the last section, we apply these results to the study of the representations $M^{(V)}$ and $S^{(V)}$ for an arbitrary (complete, non-singular) variety $V$. When $V$ is a curve of genus greater than one, our results are already known in a more explicit form, by the theory of algebraic functions of one variable.

## 1. Preliminaries.

First we explain the notations, which are used throughout this paper, and give the definitions of the representations $M^{(V)}$ and $S^{(V)}$. Let $V$ be an