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On differentiable manifolds with contact metric structures

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§ 1. Introduction.

A $(2n+1)$-dimensional differentiable manifold M^{2n+1} of class C^∞ is said to have an almost contact structure or to be an almost contact manifold if the structural group of its tangent bundle reduces to $U(n)\times 1$, where $U(n)$ means the real representation of the unitary group of n complex variables (cf. [**1**]). On the other hand, a differentiable manifold M^{2n+1} of class C^∞ is said to have (ϕ, ξ, η)-structure if there exist three tensor fields ϕ^i_j, ξ^i and η_j satisfying the relations

$$\xi^i\eta_i=1\,, \tag{1.1}$$

$$\operatorname{rank}(\phi^i_j)=2n\,, \tag{1.2}$$

$$\phi^i_j\xi^j=0\,, \tag{1.3}$$

$$\phi^i_j\eta_i=0\,, \tag{1.4}$$

$$\phi^i_j\phi^j_k=-\delta^i_k+\xi^i\eta_k\,. \tag{1.5}$$

The notions of almost contact structure and (ϕ, ξ, η)-structure are equivalent in the sense that every almost contact manifold admits a (ϕ, ξ, η)-structure and every differentiable manifold with (ϕ, ξ, η)-structure is almost contact. (cf. [2]) So, in this paper we use the word *almost contact structure* in stead of (ϕ, ξ, η)-structure.

Now, every differentiable manifold M^{2n+1} with almost contact structure admits a Riemannian metric g which satisfies the relations

$$g_{ij}\xi^j=\eta_i\,, \tag{1.6}$$

$$g_{ij}\phi^i_h\phi^j_k=g_{hk}-\eta_h\eta_k\,. \tag{1.7}$$

We call g an *associated Riemannian metric* of the almost contact structure. Although we have called the (ϕ, ξ, η)-structure with associated Riemannian metric g as the (ϕ, ξ, η, g)-structure in [2], we shall call it an *almost contact metric structure* in this paper.

By virtue of (1.1) and (1.6), ξ^i is a unit vector field. The tensor

$$\phi_{ij}=g_{ih}\phi^h_j \tag{1.8}$$