J. Math. Soc. Japan
Vol. 13, No. 4, 1961

On the quadratic norm symbol in local number fields

By Katsumi SHIRATANI

(Received April 17, 1961)

1. Introduction.

The theory of norm residue symbol in algebraic number fields has been variously treated. An explicit formula of the local norm symbol has been given as the Šafarevič symbol by Šafarevič [8]. Hasse [5] and Kneser [7] improved the result of Šafarevič and supplied a link for the 2^n-th symbols.

As for a more special case than that of Šafarevič, Yamamoto [11] has proved the local reciprocity law of Kummer-Hilbert, on which the present author gave a note [9].

The structure of norm group of Kummer extension of prime degree was characterized to a certain extent by Hensel-Hasse [6]. The specially important formula of Hasse

$$\left(\frac{\nu}{\mu}\right)\left(\frac{\mu}{\nu}\right)=(-1)^{S\left(\frac{1-\nu}{2}\cdot\frac{1-\mu}{2}\right)}, \qquad (\mu\equiv\nu\equiv 1\ (2)),$$

is widely known. Here μ, ν mean two total-positive numbers in an algebraic number field which are mutually prime, and S denotes the trace from this field to the field of rational numbers.

Recently Siegel [10] proved the formula of Hasse from the viewpoint of the Gauss-Hecke sum in the theta function theory.

In this paper it is our purpose to give a local refinement of the formula of Hasse, from which we also show that the Šafarevič-Hasse-Kneser formula [5], [7] in the quadratic case can be readily derived.

Our method is to calculate explicitly the norm elements in the quadratic case by means of an idea of Yamamoto [11] and the present author [9]. In order to make this paper self-contained we shall prove several lemmas analogous to those given in [11], [9].

2. Several preliminary lemmas.

Let k be a local number field of finite degree over the field of rational 2-adic numbers R_2, e its ramification order, f its residue class degree and k_T the field of inertia, i.e., the maximal unramified field, contained in k. We denote by $\mathfrak{O}_{R_2}$, $\mathfrak{O}_k$, $\mathfrak{O}_{k_T}$ the rings of integers in R_2, k, k_T respectively and also by $\mathfrak{l}$, π