

On the local theory of quaternionic anti-hermitian forms

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The purpose of this paper is to give a theory of the anti-hermitian forms over a p -adic quaternion algebra \mathfrak{D} , i. e. the unique quaternion division algebra over a given p -adic number field. We shall determine in §2 the types (in the sense of Witt) of such anti-hermitian forms, showing that the type of an anti-hermitian form over \mathfrak{D} is uniquely determined by the parity of the number of variables and by its discriminant, and that these two invariants can be given arbitrarily (Theorem 3). §3 is concerned with the ‘maximal integral lattice’; we shall prove the Witt decomposition theorem for such lattices, and using this, obtain some results on the structure of the group of automorphisms (or of similitudes) of an anti-hermitian form over \mathfrak{D} , which are quite analogous to those obtained by Tamagawa for other classical groups. §1 is of preliminary nature and contains some definitions and known results indispensable for our considerations.

§1. Quaternionic anti-hermitian forms and the associated sesquilinear forms.

1.1. Let k be a field of characteristic different from 2, and let \mathfrak{D} be a quaternion division algebra over k , i. e. a division algebra with a basis $(\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3)$ over k such that

$$\begin{aligned}\varepsilon_0 &= \text{identity element,} \\ \varepsilon_1^2 &= \varepsilon_0 c_1, \quad \varepsilon_2^2 = \varepsilon_0 c_2 \quad (c_1, c_2 \in k), \\ \varepsilon_1 \varepsilon_2 &= -\varepsilon_2 \varepsilon_1 = \varepsilon_3.\end{aligned}$$

For $\xi = \sum_{i=0}^3 \varepsilon_i \xi_i \in \mathfrak{D}$, the canonical involution $\xi \rightarrow \bar{\xi}$ of \mathfrak{D} is given by

$$\bar{\xi} = \varepsilon_0 \xi_0 - \sum_{i=1}^3 \varepsilon_i \xi_i$$

and the reduced norm of ξ from \mathfrak{D} to k , denoted by $n(\xi)$, is equal to $\xi \bar{\xi}$. We

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