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Remarks on differentiable structures on spheres

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J. Milnor [2] defined the invariant λ' for compact unbounded oriented differentiable $(4k-1)$-manifolds which are homotopy spheres and boundaries of π-manifolds at the same time, and proved that the invariant λ' characterizes the J-equivalence classes of these $(4k-1)$-manifolds for $k>1$. Recently S. Smale [3] has shown that a compact unbounded (oriented) differentiable n-manifold ($n\geqq 5$) having the homotopy type of S^n is homeomorphic to S^n and that two such manifolds belonging to the same J-equivalence class are diffeomorphic to each other if $n\neq 6$. Hence it turns out that the invariant λ' characterizes differentiable structures on S^{4k-1} which bound π-manifolds for $k>1$.

In this note we shall compute the invariant λ' of $B^7_{m,1}$ (S^3 bundles over S^4, see [4]) and show that every differentiable structure on S^7 can be expressed as a connected sum of $B^7_{m,1}$. We shall obtain also a similar result on S^{15}. Furthermore we shall show that $\bar{B}^8_{m,1}\cup_i D^8$ such that $m(m+1)\equiv 0 \bmod 56$ are 3-connected compact unbounded differentiable 8-manifolds with the 4th Betti number 1 and differentiable 8-manifolds of this type are exhausted by them, where $B^8_{m,1}$ are 4-cell bundles over S^4 ([4]). This will reveal that Pontrjagin numbers are not homotopy type invariants.

Notations and terminologies of this note are the same as in the previous paper [4]. We shall use them without a special reference.

1. The invariant λ' of $B^7_{m,1}$.

In the following $M_1^{n-1}\# M_2^{n-1}$ will denote the connected sum of two compact connected unbounded oriented differentiable $(n-1)$-manifolds M_1^{n-1} and M_2^{n-1} (Milnor [2]). Let W_1^n and W_2^n be two compact connected oriented differentiable n-manifolds with non-vacuous boundaries; let $f_1\colon D^{n-1}\to\partial W_1^n$ be an orientation-preserving differentiable imbedding and $f_2\colon D^{n-1}\to\partial W_2^n$ be an orientation-reversing differentiable imbedding. Then $W_1^n+W_2^n$ denotes the compact connected oriented differentiable n-manifold with boundary obtained from the disjoint union of W_1^n and W_2^n by identifying $f_1(x)$ with $f_2(x)$ ($x\in D^{n-1}$), making use of the device of "straightening the angle".

We choose an orientation of $B^7_{m,1}$ (resp. $B^{15}_{m,1}$) and that of $\bar{B}^8_{m,1}$ (resp. $\bar{B}^{16}_{m,1}$)