J. Math. Soc. Japan
Vol. 13, No. 3, 1961

On coverings of algebraic varieties

By Makoto ISHIDA

(Received Dec. 9, 1960)

Let U and V be algebraic varieties, and $f: U \rightarrow V$ a Galois covering of degree n, defined over a field k; let A and A_0 be Albanese varieties attached to U and V respectively. Then, in the preceding paper [**3**], we have proved, among several other results, the following two statements:

1) Suppose that V is embedded in some projective space. Let C be a generic hyperplane section curve on V over k and $W=f^{-1}(C)$ the inverse image of C on U; let J and J_0 be Jacobian varieties attached to (the normalization of) W and C respectively. Then the curve W generates A and we have the inequality

$$(*) \qquad \dim J - \dim A \geqq \dim J_0 - \dim A_0 .$$

2) Suppose that U and V are complete and non-singular. Then, under the assumption that the degree n is prime to the characteristic of the universal domain, the equality $\dim \mathfrak{D}_0(U) = \dim \mathfrak{D}_0(A)$ implies the equality $\dim \mathfrak{D}_0(V) = \dim \mathfrak{D}_0(A_0)$.[1]

In the present paper, we shall generalize these results to an arbitrary (i. e. not necessarily Galois) covering $f: U \rightarrow V$. Moreover, the result 2) will be replaced by a better one, i. e. the inequality

$$(**) \qquad \dim \mathfrak{D}_0(U) - \dim \mathfrak{D}_0(A) \geqq \dim \mathfrak{D}_0(V) - \dim \mathfrak{D}_0(A_0) .$$

Here we note that the numbers on the both sides of (*) and (**) are non-negative (cf. Lang [**4**] and Igusa [**1**]) and that the assumption on the degree n in (**) is essential as easily seen in Igusa [**2**]. It seems to be worth noting that the inequalities (*) and (**) may be rewritten in the following forms:

$$(*)' \qquad \dim J - \dim J_0 \geqq \dim A - \dim A_0 .$$

$$(**)' \qquad \dim \mathfrak{D}_0(U) - \dim \mathfrak{D}_0(V) \geqq \dim \mathfrak{D}_0(A) - \dim \mathfrak{D}_0(A_0) .$$

The numbers on the both sides of (*)′ and (**)′ are also non-negative. As in [**3**], using the formula of Hurwitz on the genera of curves, we can deduce from (*)′ an estimation of the irregularity of the covering variety U of V. In addition to these two inequalities, we shall prove, for this arbitrary covering $f: U \rightarrow V$, some analogous results to the main theorems in [**3**].

1) For a complete, non-singular variety W, we donote by $\mathfrak{D}_0(W)$ the space of the linear differential forms of the first kind on W.