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Remarks on Cantor's Absolute

By Gaisi TAKEUTI

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The purpose of this paper is to prove a theorem which is stated in the introduction as the main theorem. In this paper it is understood that a set theory means a set theory T in the first order predicate calculus satisfying the following conditions:

1) $\in$ is the only predicate in T. ($a=b$ is an abbreviation of $\forall x(x\in a \leftrightarrow x\in b)$).
2) T is a consistent extension of Zermelo-Fraenkel's set theory.

A model $\langle A, \overset{*}{\underset{A}{\in}} \rangle$ of a set theory is called 'regular', if and only if there exists no (infinite) sequence $a_0, a_1, a_2, \cdots$ of elements of A such that

$$a_1 \in {}^{*}_{A} a_0,\ a_2 \in {}^{*}_{A} a_1,\ a_2 \in {}^{*}_{A} a_2, \cdots$$

hold. Here a sequence is understood in the informal sense; it may be undefinable in any way.

We presuppose that there exists something absolute, which is a vast universe consisting of numerous concrete sets, and in which some properties (in the informal sense) are "well-defined". Such a universe C will be called *Cantor's Absolute.* It should be understood as a transcendental existence. An existencial quantifier $\exists x$ and universal quantifier $\forall x$ mean *literally* "there exists a set x such that $\cdots$" resp. "for every set x, it holds that $\cdots$". A closed formula in which $\in$ only is used as predicate, is *a priori* true or false in Cantor's Absolute.

Moreover the following propositions are assumed to hold.

(1) Let T_C be the class of all true closed formulas in Cantor's Absolute consisting solely of logical symbols, the predicate $\in$ and bound variables. T_C is called Cantor's set theory. Then T_C contains the class of all provable closed formulas in the set theory of Zermelo-Fraenkel.
(2) $\langle C, \in \rangle$ is a regular model of T_C.
(3) For any well-defined property and any set a in C, there exists a set consisting of all sets which belong to a and satisfy the property. (The word 'property' is used in the informal sense.)

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