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On some relations between the Martin boundary and the Feller boundary

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1. In this paper we shall consider the integral representation of bounded harmonic functions by means of a regular Borel measure on the Feller boundary $\mathcal{M}(\mathfrak{S})$ (cf. Section 9). For this purpose we investigate mutual relations between the family of bounded harmonic functions, a function lattice on the Martin boundary and a function lattice on the Feller boundary, by use of the Martin representation theorem of harmonic functions (cf. J. L. Doob [**3**] and T. Watanabe [**12**], [**13**]). This subject is closely related to some results of D. G. Kendall [**9**] which we shall prove here by a different method.

2. Let X be a countable state space with the discrete topology. Let $X \cup \{\rho\}$ be denoted by $\tilde{X}$ in which $\{\rho\}$ is added to X as an isolated point. Let W be the totality of $\tilde{X}$-valued right-continuous functions w on the interval $T=[0, \infty]$. The value of w at time t is denoted by $w(t)$ or $x_t(w)$. Let $\mathbf{M} = \{X, W, P_x, x \in \tilde{X}\}$ be a minimal Markov process[1] where X is the state space, W is the sample space and P_x is the probability measure on the Borel field $\mathcal{F}(W)$ generated by the sets $\{w\,; x_t(w) \in A\}$ (A : a Borel set on $\tilde{X}$). Define

$$\begin{aligned} \sigma_A(w) &= \inf\{t>0\,; x_t(w) \in A\} && \text{if } x_t(w) \in A \text{ for some } t>0\,, \\ &= +\infty && \text{otherwise,} \\ \tau_A(w) &= \inf\{t>0\,; x_t(w) \notin A\} && \text{if } x_t(w) \notin A \text{ for some } t>0\,, \\ &= +\infty && \text{otherwise.}^{2)} \end{aligned}$$

For $x, y \in \tilde{X}$, we set $\Pi(x, y) = P_x\{w\,; x_{\tau_x}(w) = y,\ \tau_x < +\infty\}$. Then $\Pi(x, \rho) = 1 - \sum_{y \in X} \Pi(x, y)$ and $\Pi(\rho, \rho) = 1$.

In this paper, a finite real valued function $u(\cdot)$ over X will be called x_t-harmonic if it satisfies $u(x) = \sum_{y \in X} \Pi(x, y) u(y)$ (in the sense of absolute convergence) for any x in X.

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1) The term 'minimal process' is used in the sense of W. Feller [**6**, pp. 535-537]. Also a precise definition of such process is seen in [**13**, Chapter 1].

2) We denote τ_x in case $A = \{x\}$.