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On automorphisms of conformally flat K-spaces

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Introduction. It is known that in a compact almost-Kählerian space an infinitesimal isometry is almost-analytic and hence an automorphism.[1] On the other hand, in a compact K-space an infinitesimal isometry is not necessarily an automorphism.[2] In the 6-dimensional unit sphere with the structure given by Fukami-Ishihara, which is an example of a compact K-space, an almost-analytic transformation is an isometry and hence is an automorphism.[3]

In this paper we shall give some theorems on the automorphisms of conformally flat K-spaces.

In §1 we shall give definitions and well known identities. In §2 we shall deal with a conformally flat K-space and prove that the scalar curvature of such a space is non-negative constant. In §3 we shall obtain a theorem on automorphisms of compact conformally flat K-spaces. The last section will be devoted to discussions on automorphisms of K-spaces of positive constant curvature.

1. Preliminaries. Let us consider an n-dimensional K-space M.[4] By definition, M admits a tensor field $\varphi_i{}^h$ and a positive definite Riemannian metric tensor g_{ji} such that

$$\varphi_i{}^r\varphi_r{}^h = -\delta_i^h, \tag{1.1}$$

$$g_{rs}\varphi_j{}^r\varphi_i{}^s = g_{ji}, \tag{1.2}$$

$$\nabla_j\varphi_i{}^h = -\nabla_i\varphi_j{}^h, \tag{1.3}$$

where ∇ denotes the operator of Riemannian covariant derivation.

(1.1) and (1.2) mean that M is an almost-Hermitian space and hence is even dimensional and orientable.

The tensor $\varphi_{ji} = \varphi_j{}^r g_{ri}$ is skew-symmetric by virtue of (1.1) and (1.2) and so is $\nabla_j\varphi_{ih}$ by (1.3). φ_{ji} is a Killing tensor of order 2 in the sense of Yano-Bochner [**6**].

1) Tachibana, S., [**2**]. The number in brackets refers to Bibliography at the end of the paper.

2) Tachibana, S., [**3**].

3) Fukami, T. and S. Ishihara., [**1**].

4) As to the notations we follow Tachibana, S., [**3**]. Indices run over $1, 2, \cdots, n$. Throughout the paper we assume that $n > 2$.