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On almost-analytic tensors of mixed type in a K-space

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§ 0. Introduction.

Let X_n be an n-dimensional differentiable manifold with local coordinates $\{x^i\}$[1]. On this manifold a tensor field $\varphi_j{}^i$ such that

$$\varphi_r{}^i\varphi_j{}^r = -\delta_j^i \tag{0.1}$$

is called an almost-complex structure and a differentiable manifold X_n with such an almost-complex structure is called an almost-complex manifold or an almost-complex space[2].

An almost-complex space X_n with an almost-complex structure satisfying

$$g_{rs}\varphi_j{}^r\varphi_i{}^s = g_{ji} \tag{0.2}$$

where g_{ji} is a positive definite Riemannian metric tensor is called an almost-Hermitian space[3]. In this place, it is easily verified that $\varphi_{ji} = -\varphi_{ij}$ where $\varphi_{ji} = \varphi_j{}^r g_{ri}$.

On the other hand, A. Frölicher[4] proved that there exists an almost-complex structure on the six dimensional sphere S^6, and T. Fukami and S. Ishihara[5] proved that the structure on S^6 is an almost-Hermitian one satisfying

$$\nabla_j\varphi_{ih} + \nabla_i\varphi_{jh} = 0 \tag{0.3}$$

where ∇_j denotes the operator of covariant derivation with respect to the Riemannian connection.

In this paper, by a K-space[6] we shall always mean an n-dimensional almost-Hermitian space satisfying the condition (0.3).

Now, a necessary and sufficient condition that in a compact K-space a vector be almost-analytic (see § 1) has been obtained for a contravariant vector by S. Tachibana in [**10**] and for a covariant vector by the author in [**7**].

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1) Through this paper the Latin indices run over the values $1, 2, \cdots, n$.
2), 3) K. Yano [**13**, p. 228].
4) A. Frölicher [**3**].
5) T. Fukami and S. Ishihara [**4**].
6) S. Tachibana [**10**].