J. Math. Soc. Japan
Vol. 13, No. 2, 1961

# Bonded groups

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(Received Dec. 14, 1959)

## § 1. Introduction.

Historically, discrete flows and continuous flows have played the most important roles in topological dynamics (see [**1**] and [**2**]). A *discrete flow* is a transformation group whose phase group is the additive group $I$ of all integers with the discrete topology. A *continuous flow* is a transformation group whose phase group is the additive group $R$ of all real numbers with the usual topology. As we know, in either $R$ or $I$, every non-trivial cyclic subgroup (i. e. a cyclic group generated by a non-identity element of the group) is syndetic (we call this Property S). A subset $N$ of a topological group $G$ is called left syndetic (see [**2**]) if there exists a compact subset $K$ of $G$ such that $N\cdot K=NK=\{xy\mid x\in N, y\in K\}=G$. Similarly, we can define *right syndetic subsets*. A set $N$ is called *syndetic* if it is both left syndetic and right syndetic. A subgroup $H$ of $G$ which is left syndetic is also right syndetic, and vice versa. As we know, the almost periodicity properties of transformation groups are based on syndetic subsets of the phase group. It is interesting to consider the following problem:

"What is the structure of a topological group which has the Property S?" The author will show in this paper (see Theorem 4) that a group of this type is either (a) compact, (b) topologically isomorphic to $I$, (c) topologically isomorphic to $R$ or (d) radical (see [**7**]) which is not locally compact (see Theorem 5).

We rarely consider compact transformation groups, ab initio, in topological dynamics, since under a compact phase group each point is always almost periodic (see [**2**]) and recurrent (see [**2**]) and any orbit is equal to its orbit closure. However, it is interesting we discover a new type group, the non-locally compact, radical group having the Property S.

In the present paper, a topological group will be denoted by $G$, and $e$ (0 in the abelian case) will denote either the identity element of $G$ or the trivial subgroup consisting of the identity only. The additive group of all integers with the discrete topology will be denoted by $I$, and the additive group of all real numbers with the usual topology will be denoted by $R$. A *topological isomorphism* between two topological groups, $G_1$ and $G_2$, is simultaneously an