Journal of the Mathematical Society of Japan Vol. 12, No. 4. October, 1960

On a smoothing operator for the wave equation.

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(Received Dec. 11, 1959)

1. Introduction. The Cauchy problem for the classical wave equation

$$\square_n u=\left(\frac{\partial^2}{\partial x_0{}^2}-\sum_{i=1}^{n-1}\frac{\partial^2}{\partial x_i{}^2}\right)u=f(x)$$

has been the subject of many investigations for more than half a century. During this time several formulas have been deduced for the solution of this problem with the Cauchy data given on the plane $x_0=0$. The difficulties in obtaining explicit formulas all center around the fact that the direct methods of integration lead to singular integrals. These difficulties have been overcome by various methods which either avoid singular integrals or select the appropriate "part" of such integrals. Among the latter methods the best known is that introduced by Hadamard and developed in his lectures on the Cauchy problem [6] (numbers in brackets refer to the bibliography at the end of the paper). This work has been extended by Bureau in a number of papers (see, for example, [7], [8]). Among the methods which seek to avoid singular integrals the most recent seem to be those of Weinstein [9], Diaz and Martin [10] and M. Riesz [1]. The work of Riesz on the wave equation has been extended by Garding [2] to the class of linear hyperbolic equations with constant coefficients. This method depends on the analytic continuation of certain integrals with respect to a complex parameter. Subsequently Leray [3] generalizing the Riesz-Garding method, showed that smoothing operators could be introduced in order to avoid singular integrals and re-derived the Riesz formulas for the wave equation.

In this paper we shall show that a suitable smoothing operator for the wave equation is simply $\partial^m/\partial x_0{}^m$ for m properly chosen and derive new formulas for the solution of the Cauchy problem with data given on the plane $x_0=0$. At the same time we shall show that it is not difficult to construct a solution of the equation

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1) Work on this paper was supported, in part, by the Office of Naval Research. The author wishes to thank Professor P. C. Rosenbloom of the University of Minnesota for several stimulating conversations on this subject which resulted in the successful completion of the work.