Journal of the Mathematical Society of Japan Vol. 12, No. 4, October, 1960

Ordinal Diagrams II.

By Gaisi TAKEUTI

(Received Nov. 12, 1959)

In a former paper [1] the author developed the theory of ordinal diagrams, which represent the ordinal numbers in a certain "Abschnitt" of the second number class and are useful for the consistency proof of some logical systems. In this paper we shall generalize the notion of ordinal diagrams and their ordering relations, and we shall prove that the generalized system $\mathrm{Od}(I, A, S)$ of ordinal diagrams is well-ordered.

The well-ordering property of the system of ordinal diagrams will be lost if we generalize the notion of ordinal diagrams in the following directions:

a) Making use of an ordinal diagram in place of $i \in I$.

b) Making use of an ordinal diagram in place of $s \in S$.

In fact, we shall have, in case of a), a strictly descending sequence:

$1 > (1, 0, 0) > ((1, 0, 0), 0, 0) > \cdots$

and in case of b),

$1 > (0, 1) > (0, (0, 1)) > \cdots$

Most of expressions in this paper should be read and understood according to the context.

§ 1. Ordinal diagrams constructed from I, A and S.

1. Let I, A and S be well-ordered sets. The system $\mathrm{Od}(I, A, S)$, called the *system of ordinal diagrams constructed from* I, A and S, is defined recursively by means of the operations of $(\cdot, \cdot), (\cdot, \cdot, \cdot)$ and $\#$ as follows. (The word "ordinal diagram" or "o.d." which was used in [1] is now applied to "element of $\mathrm{Od}(I, A, S)$." If no confusion is feared, the same symbol $<$ is used to denote the order of I or A or S; the symbol $<$, as well as the equality $=$, should be understood according to the context.)

1.1. If $a \in A$, then a is an o.d.

1.2. If α is an o.d. and $s \in S$, then (α, s) is an o.d.

1.3. If α and β are o.d.'s and $i \in I$, then (i, α, β) is an o.d.

This work was done under Appointment supported by the International Cooperation Administration under the Visting Research Scientists Program administered by the National Academy of Science of the United States of America.