

On affinely connected manifolds admitting groups of affine motions with complex reducible linear isotropy groups.

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With respect to affinely connected manifolds admitting groups of affine motions of various types and with respect to the groups themselves, especially on their dimensions, there are many papers, for instance, by I. P. Egorov, H. C. Wang and K. Yano [5], Y. Mutō [6] and the others.

In this paper, we study affinely connected manifolds admitting groups of affine motions of some types with complex reducible linear isotropy groups, that is, with linear isotropy groups which are real representations of complex linear homogeneous groups.

The main purpose is to prove Theorems 4.1, 4.2 and 4.3 in § 4, as applications of Theorem 3.1 and Corollary 3.1 in § 3.

§ 1. Preliminary remarks.

The notations $GL(n, R)$, $GL(m, C)$, $SL(n, R)$, $SL(m, C)$ are as usual and furthermore we denote the real representations of $GL(m, C)$ and $SL(m, C)$ by $RGL(m, C)$ and $RSL(m, C)$ respectively. The other notations are as follows.

E_N : unit matrix of degree N .

H^1 : real one dimensional homothetic group: $x \rightarrow rx$ (x, r : real; $r > 0$).

H_N : real one dimensional group composed of all $(N \times N)$ -matrices aE_N (a : positive real).

T^1 : one dimensional torus group: $z \rightarrow \sigma z$ (σ, z : complex; $|\sigma| = 1$).

T_m : one dimensional group composed of all complex $(m \times m)$ -matrices σE_m (σ : complex; $|\sigma| = 1$).

$R(T_m)$: real representation of T_m .

A_{2m} : $2m$ -dimensional affinely connected manifold of class C^∞ .

G : Lie group of affine motions of A_{2m} .

$G(P)$: isotropy group of G leaving invariant a generic point P of A_{2m} .

$G_0(P)$: linear isotropy group of G at a generic point P , which is the faithful linear representation of $G(P)$. We mean the connected component of the identity.