Journal of the Mathematical Society of Japan Vol. 12, No. 4, October, 1960

On the Goldbach problem in an algebraic number field II.

By Takayoshi MITSUI

(Received Aug. 18, 1959)

§ 4. Treatment of $I_s(\mu;\lambda)$ (I).

Let λ be a totally positive integer with sufficiently large norm $N(\lambda)$ and $\Omega(\lambda)$ be the set of all prime numbers ω such that

$$(4.1)\qquad \begin{aligned} &0<\omega^{(q)}\leqq\lambda^{(q)} && (q=1,2,\cdots,r_1),\\ &|\omega^{(p)}|\leqq|\lambda^{(p)}| && (p=r_1+1,\cdots,r_1+r_2). \end{aligned}$$

We shall define a trigonometrical sum

$$(4.2)\qquad S(z;\lambda)=\sum_{\omega\in\Omega(\lambda)}e^{2\pi iS(\omega z)},$$

where $z=(z_1,z_2,\cdots,z_n)$ is a point of E.

We know by (2.1) that $z_1,z_2,\cdots,z_n$ are written in the form

$$z_j=\sum_{k=1}^{n}x_k\delta_k^{(j)}\qquad (j=1,2,\cdots,n)$$

with real numbers $x_1,x_2,\cdots,x_n$. Taking $x_1,x_2,\cdots,x_n$ as variables, we consider an integral

$$(4.3)\qquad I_s(\mu:\lambda)=\int_{-1/2}^{1/2}\!\!\cdots\!\int S(z;\lambda)^s e^{-2\pi iS(\mu z)}dx_1dx_2\cdots dx_n,$$

where s is a positive rational integer, μ is a totally positive integer and the domain of integration is given by the conditions

$$|x_j|\leqq\frac{1}{2}\qquad (j=1,2,\cdots,n).$$

We see that $I_s(\mu;\lambda)$ is equal to the number of the s-tuples $(\omega_1,\omega_2,\cdots,\omega_s)$ of prime numbers which satisfy the following conditions:

$$\mu=\omega_1+\omega_2+\cdots+\omega_s,$$
$$\omega_j\in\Omega(\lambda)\qquad (j=1,2,\cdots,s).$$

Therefore, for any totally positive unit η we have

$$I_s(\eta\mu;\eta\lambda)=I_s(\mu;\lambda).$$

On the other hand, by suitable choice of a totally positive unit η_0 we have

$$c_1N(\lambda)^{1/n}<|\lambda^{(j)}\eta_0^{(j)}|<c_2N(\lambda)^{1/n}\qquad (j=1,2,\cdots,n).$$