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On vector differential forms attached to automorphic forms.

Dedicated to Professor Z. Suetuna.

By Michio KUGA and Goro SHIMURA

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In recent works [2], [3], it was found that the integral of certain vector differential forms, attached to automorphic forms with respect to a Fuchsian group G, is important in the arithmetic theory of modular correspondences. Those vector differential forms ω are defined on the upper half plane and satisfy the transformation formula

$$\omega\circ\sigma=M(\sigma)\omega \tag{1}$$

for every element σ of the group G, where $M(\sigma)$ is a tensor representation of G. The object of the present paper is to determine all holomorphic forms satisfying this relation (1). M being of degree $2m-1$, we can attach to every cusp form of degree $\leqq 2m$ a holomorphic form ω with the representation M (Theorem 1). Conversely, any holomorphic form satisfying (1) is expressed as a sum of the forms thus obtained from cusp forms of degree $\leqq 2m$; and this expression gives a direct decomposition of the vector space $\mathfrak{F}$ of such holomorphic forms (Theorem 2). Hence the dimension of the vector space $\mathfrak{F}$ is easily obtained if we know the dimension of the linear space of cusp forms for each degree. We note that the integral of the form attached to a cusp form of degree $<2m$ has a period cohomologous to 0, in the sense described in [3]. This fact distinguishes among such forms the forms attached to cusp forms of degree $2m$, which were the object of the investigation in [3].

§ 1. Cusp forms with respect to a Fuchsian group.

Let $\mathcal{H}$ denote the upper half plane, the set of all complex numbers with positive imaginary parts. Every element $\sigma=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of $SL(2, \boldsymbol{R})$ operates on $\mathcal{H}$, as usual:

$$\sigma(z)=\frac{az+b}{cz+d};$$

we put

$$J(\sigma, z)=(cz+d)^{-1}.$$