Journal of the Mathematical Society of Japan Vol. 12, No. 3, July, 1960

Commutative group varieties.

To the memory of Y. Taniyama.

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(Received Nov. 29, 1958)

The purpose of this paper is to generalize some results of Weil [**6**] on abelian varieties to the case of commutative group varieties. An element, of a group, whose order is finite and divides n will be called an *n-division point*. In § 1, we first count the number of n-division points on a commutative group variety and see that a commutative group variety without affine subgroup is generated by division points. We can introduce therefore a system of l-adic coordinates on such a group G, and get the l-adic representation of the ring of endomorphisms of G. Next we shall show the symmetric property of isogenies between divisible commutative group varieties, where an isogeny means a surjective (rational) homomorphism between two group varieties of the same dimension. In § 2, we shall see that a group variety defined over a finite field is generated by an abelian variety and a linear group variety (Theorem 1), and that the algebra of endomorphisms of a divisible commutative group variety defined over a finite field is a semi-simple algebra over the field of rational numbers.

We use the following terminologies and notations throughout the paper. A homomorphism of a group variety into a group variety means always a rational homomorphism; we use " endomorphism " in the corresponding sense. An algebraic subgroup of a group variety is an abstract subgroup which is a closed subset in the sense of Zariski topology. An affine group is a group variety which is biregularly equivalent to an affine space as a variety. G_a denotes the additive group of the universal domain and G_m the multiplicative group of the non-zero elements of the universal domain. A biregular isomorphism between group varieties is a group-isomorphism defined by a birational mapping which we denote by $\cong$. $T \supset S$ means that T contains S but not equals S. For a natural number n, we denote by $n[G]$ the number of n-division points on a group G. We denote the characteristic of the universal domain by p. We write the (commutative) group-operation additively.

§ 1. Division points.

1. Let G be a commutative group variety and L be its maximal linear