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A probabilistic method in Hausdorff moment problem and Laplace-Stieltjes transform.

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§ 0. Introduction.

The purpose of this paper is to show that the proofs of some main representation theorems for moment sequences and Laplace-Stieltjes transforms are obtained probabilistically by making use of the representation theorems in the theory of Martin boundaries induced by Markov processes [6].

We prepare, in § 1, the following three topics: (a) the definition of time discrete Markov processes over the denumerable space, (b) the definition and some properties of process harmonic functions and (c) the summary of the theory of Martin boundaries for the above processes. In § 2 we shall construct the Martin boundary for the space-time Markov process attached to the Bernoulli sequence B(1/2). In § 3, using the results of § 2, we shall derive the representation theory for moment sequences which is known as Hausdorff moment problem.

In the last section we shall discuss the representation theory for Laplace-Stieltjes transforms in connection with the space-time Markov process attached to the standard Poisson process. This may be considered as a continuous analogue of § 1 through § 3.

It may be interesting to apply the above method to the representation theory for more general transforms with positive kernels. For example, the convolution transforms [2] are expected to have a close relation with the space-time Markov processes attached to additive processes.

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§ 1. The sketch of the theory of Martin boundaries induced by time descrete Markov processes over the denumerable space.

All the results in this section are stated without proof: they shall be discussed systematically in [6].

Let E be the denumerable space $\{1, 2, 3, \cdots\}$ with the discrete topology and an extra point ∞ be added to E as an isolated point. We shall denote